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J. Phys.: Condens. Matter **20** (2008) 204134 (5pp) doi:10.1088/0953-8984/20/20/204134

Numerical investigation of thermomagnetic convection in a heated cylinder under the magnetic field of a solenoid

D Zablockis, V Frishfelds and E Blums

Institute of Physics of University of Latvia, 32 Miera street, LV-2169 Salaspils, Latvia

E-mail: dmitrijs.zablockis@gmail.com

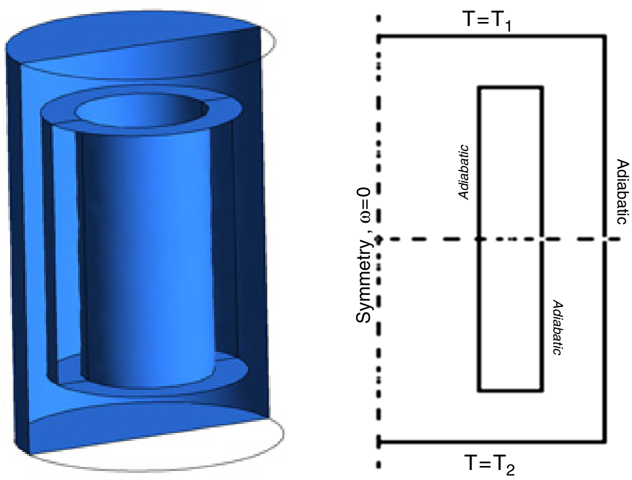
Received 3 April 2008
Published 1 May 2008
Online at stacks.iop.org/JPhysCM/20/204134

Abstract
The main objective of this paper is the numerical investigation of the process of thermomagnetic convection of a special temperature sensitive ferrofluid. The fluid is studied in a cylindrical domain, with constant temperatures on the top and bottom ends and adiabatic boundary conditions on the sidewalls. The thermomagnetic convection is generated by a non-uniform constant magnetic field of a solenoid, which is placed in a hollow area inside the domain. It has been found that the efficiency of convective heat transfer in such a set-up can be increased up to sevenfold by magnetic field within the studied range of parameters.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The effect of thermomagnetic convection in ferrofluids shows promising possibilities for applications, particularly in miniature cooling devices for electronics and in low-gravity environments where natural convection fails to provide adequate heat transfer. A number of previous theoretical and experimental studies (some of recent results— [1, 2, 4, 5]) have shown that under certain conditions non-uniform magnetic fields facilitate heat transfer in ferrofluids. Thermosyphon-type cooling devices based on this effect could be self-regulating, self-sustaining and considerably smaller than active set-ups. Our aim is to numerically study the thermomagnetic convection in a simplified cylindrical set-up and determine the achievable efficiency of heat transfer in a thermosyphon-type device.



(a) (b)

Figure 1. Geometry of the problem: (a) cylindrical enclosure with coaxially positioned solenoid, (b) 2D computational domain.

2. Modelled system and equations

2.1. Modelled domain

The modelled geometry (figure 1) is a 3D cylindrical enclosure with height/radius aspect ratio equal to three. The solenoid is positioned coaxially inside the enclosure. It has inner and outer radii equal to one-half and three-quarters of the cylinder radius. The height of the solenoid is five-sixths of the cylinder height. The geometry and boundary conditions are axially

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symmetric, which allows us to reduce the problem to 2D. The modelled domain, therefore, consists of a rectangular region, which includes half of the cylinder's axial cross-section.

2.2. Governing equations

The magnetic fields in question are sufficiently strong to be considered constant and all perturbations due to the fluid motion can be neglected. The field produced by the solenoid can thus be determined using the vector potential formulation of the magnetostatics equations

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{J}. \tag{1}$$

The flow of the magnetic fluid is governed by the Navier–Stokes equation with buoyancy and Kelvin's body force terms, continuity equation, temperature equation and equations of state. Utilizing the Boussinesq approximation they are respectively

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\, \mathbf{V} \right) = -\nabla P + \eta \Delta \mathbf{V} + \rho \mathbf{g} + \mathbf{M}\nabla \mathbf{B}, \tag{2}$$

$$\nabla \mathbf{V} = 0, \tag{3}$$

$$\frac{\partial T}{\partial t} + \mathbf{V}\nabla T = \chi \Delta T, \tag{4}$$

$$\rho = \rho_0 \left(1 - \beta T\right), \qquad \mathbf{M} = \mathbf{M}_0 \left(1 - K\Theta\right). \tag{5}$$

Linear dependences have been chosen to approximate the temperature dependence of density and magnetic moment (5). The potential parts of magnetic and gravity force have been included in the pressure gradient [2]. Introducing the dimensionless parameters *Ra*—Rayleigh number, *Rm*—magnetic Rayleigh number—and *Pr*—Prandtl number—allows us to write the equations in the more compact dimensionless form. Due to the 2D nature of the flow, the problem can also be further simplified, by transforming the equations to the vorticity-stream function formulation [3]

$$\frac{\partial \boldsymbol{\omega}}{\partial \tau} + (\mathbf{v}\nabla)\, \boldsymbol{\omega} = Pr\Delta\boldsymbol{\omega} - Ra\,Pr\nabla \times (\theta \mathbf{g}) - Rm\,Pr\nabla \times (\theta \mathbf{m}\nabla \mathbf{b}), \tag{6}$$

$$\Delta \boldsymbol{\psi} = -\boldsymbol{\omega}, \tag{7}$$

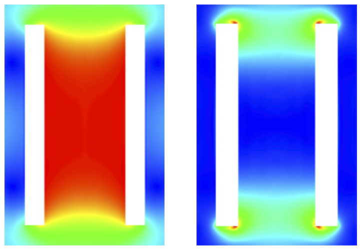
$$\frac{\partial \theta}{\partial \tau} + (\mathbf{v}\nabla)\theta = \Delta\theta. \tag{8}$$

In the 2D case the only non-zero components of the vector potential, stream function and vorticity vectors are those normal to the axial cross-section. The appropriate boundary conditions for the problem include zero vector potential on the symmetry axis and a mixed type boundary condition on the outer surface representing the solenoid in the dipole approximation. The stream function has been set to zero on all the outer boundaries and the symmetry axis. In order to obtain the value of the stream function on the inner boundary (the surface of the solenoid) the Navier–Stokes equation in the pressure–velocity formulation has been explicitly integrated across that boundary. The velocity is zero because of the imposed non-slip boundary conditions and an integral over a closed contour of tangential pressure gradient is also zero

$$0 = \oint \left(\nabla \times \boldsymbol{\omega} + Ra\theta \mathbf{g} + Rm\theta \left(\mathbf{m}\nabla\right) \mathbf{b} \right). \tag{9}$$

This equation is then added to the system. The condition $\boldsymbol{\omega} = 0$ is set on the symmetry axis, no-slip boundary conditions for vorticity have been imposed on the outer boundary and the surface of the solenoid [3]. Zero-flux conditions for temperature are set on the outer sidewall of the domain and on the symmetry axis. Constant temperatures are defined on the upper and lower ends of the cylinder. The surface of the solenoid is thermally isolated.

This set of equations (6)–(9) and the boundary conditions have been written in the finite difference (FD) formulation using the upwind scheme for the advection term. The resulting linear system has been solved numerically on a non-uniform non-staggered grid by an iterative Bi-CG solver.



(a) (b)

Figure 2. (a) Magnetic field configuration produced in the cylinder by a solenoid; (b) interpolation of ∇B^2, representing the areas of maximum magnetic pressure.

3. Results and discussion

Initially, a series of calculations was performed to study the distribution of the magnetic field and magnetic force acting on the ferrofluid for different configurations of the solenoid and the enclosure. A typical distribution of the computed magnetic field is shown in figure 2(a) and the corresponding interpolation of ∇B^2, which governs the magnetic force acting on the ferrofluid, is given in figure 2(b). An appropriate geometry of the domain and the solenoid was then chosen in order to position the areas where the fluid is cooled or heated (top and bottom surfaces of the enclosure) inside the magnetic field.

Assuming the distribution of the magnetic field as a constant parameter, transient development of the stream function, vorticity and temperature fields has been calculated for a range of Rayleigh and magnetic Rayleigh numbers. The parameters used in the calculations are summarized in table 1. The Prandtl number in this case is 10 and *Rm* and *Ra* can be adjusted, changing accordingly the magnetic field and the temperature gradient. Of main interest were the stationary cases, which developed when the dimensionless time approached unity. First, a series of calculations was performed to study the ferrofluid flow at zero magnetic Rayleigh number ($Rm = 0$), which corresponds to pure thermal convection,

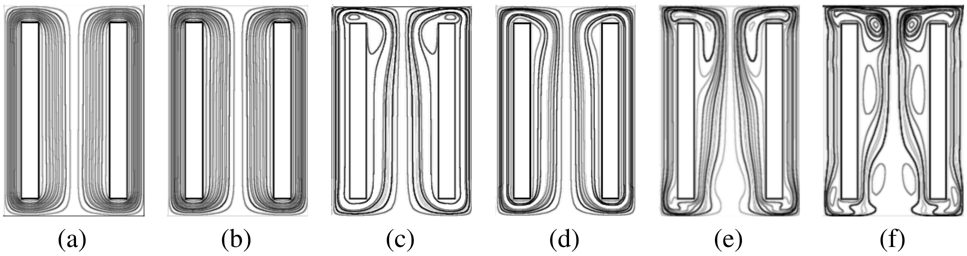


Figure 3. Stream function contours for $Ra = 10^4$ and different values of Rm/Ra: (a) $Rm/Ra = 0$, (b) $Rm/Ra = 2.5$, (c) $Rm/Ra = 40$, (d) $Rm/Ra = 100$, (e) $Rm/Ra = 200$, (f) $Rm/Ra = 1000$. The applied temperature gradient is oriented downwards, $\nabla T \downarrow$.

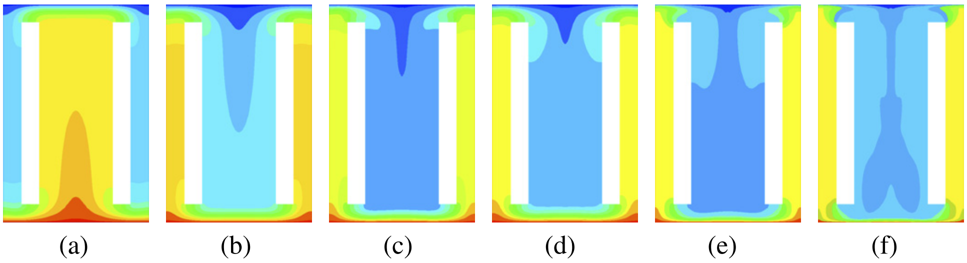


Figure 4. Temperature distribution contours for $Ra = 10^4$ and different values of Rm/Ra: (a) $Rm/Ra = 0$, (b) $Rm/Ra = 2.5$, (c) $Rm/Ra = 40$, (d) $Rm/Ra = 100$, (e) $Rm/Ra = 200$, (f) $Rm/Ra = 1000$. The applied temperature gradient is oriented downwards, $\nabla T \downarrow$.

Table 1. Parameters related to the magnetic fluid in question.

Parameter	Value
Viscosity, η	10^{-3} kg m^{-1} s^{-1}
Density, ρ	10^3 kg m^{-3}
Thermal diffusivity, κ	10^{-7} m^2 s^{-1}
Thermal expansivity, β	5×10^{-4} K^{-1}
Concentration, ϕ	0.02
Saturation magnetization, M_S	5×10^6 A m^{-1}
Pyromagnetic coefficient, K	5×10^{-3} K^{-1}
Particle radius, r	5 nm

varying only the temperature difference between the upper and lower borders of the domain. The temperature gradient was directed downwards in this case. The structure of the resulting flow includes an upgoing warm stream due to buoyancy inside the inactive solenoid and downgoing cold flow at the outer areas. A contour plot of the corresponding typical stream function is shown in figure 3(a) and the temperature distribution in figure 4(a) for a particular case of $Ra = 10^4$. The Rayleigh number was varied in a broad interval between 10^3 and 10^7. At Rayleigh numbers higher than 10^7 the flow exhibits undamped transient pulsations, which become turbulent, further increasing Ra. To study the influence of the magnetic field on convection we changed the relation Rm/Ra for different Rayleigh numbers. The presence of a sufficiently strong magnetic field (when Rm becomes comparable with Ra) changes the structure of the flow. The area inside the solenoid in this case is occupied by the downgoing cold fluid, pulled by the magnetic field. The temperature profiles are shown in figures 4(b)–(f) and the corresponding stream function contour plots are given in figures 3(b)–(f) for the case of $Ra = 10^4$. Increasing the strength of the magnetic field, new circulation areas appear at the lower end of the solenoid, inside the solenoid and at the top end.

The effects of thermomagnetic convection on the heat transfer have been characterized by the relative Nusselt number (normalized by the Nusselt number in case of pure thermal convection for the same Rayleigh number) in each case. The time development curves of this quantity for a single case of $Ra = 2 \times 10^5$ and multiple values of the factor Rm/Ra are shown in figure 5. Stationary states have been achieved in all cases. The dependence of the relative Nusselt number on the factor Rm/Ra for various values of the Rayleigh number are shown in figure 6. The curves have a sharp bend at $Rm/Ra \approx 10$, which corresponds to an instability similar to the Rayleigh instability.

The results show that heat transfer in the case of thermomagnetic convection can be up to seven times more efficient than in the case of pure natural convection. The dependence on figure 6 agrees well with the scaling analysis by Mukhopadhyay *et al* [4], although it has been performed for a slightly different system. The region at higher magnetic fields can be approximated by a power-law, with the exponent ≈ 0.3.

Another interesting case is when the temperature gradient is oriented upwards and the thermomagnetic convection is not aided by buoyancy. A series of calculations for different Ra and Rm/Ra values has been performed to study this situation. Variation of Rm/Ra for a selected value of the Rayleigh number ($Ra = 10^4$) is represented in figure 7. The central area is occupied by an upgoing cold stream, pulled inside the solenoid by the magnetic field. The corresponding temperature distribution is given in figure 8. The efficiency of the heat transfer in this case is also characterized by the relative Nusselt number. Its dependence on Rm/Ra for the values of Ra in the interval between 2×10^2 and 10^6 is represented

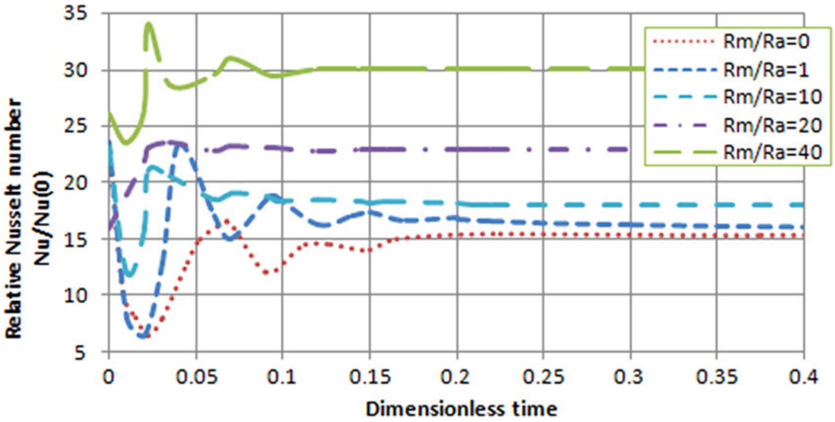


Figure 5. Time development of convective heat transfer. $\nabla T \downarrow$.

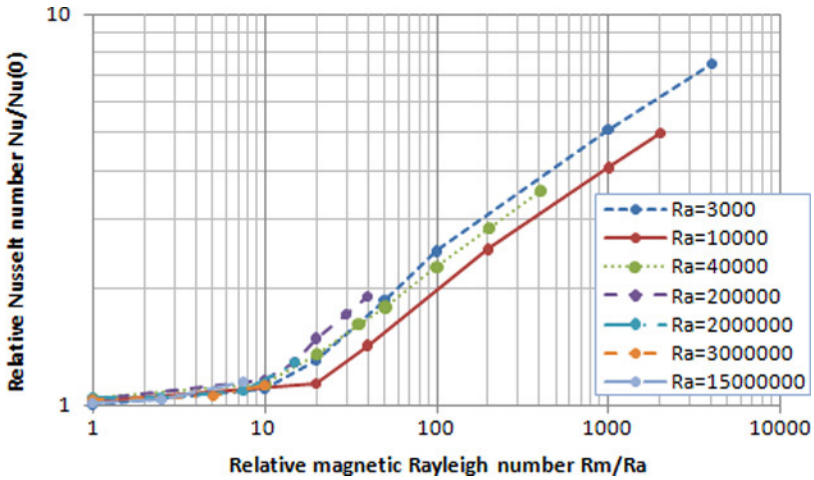
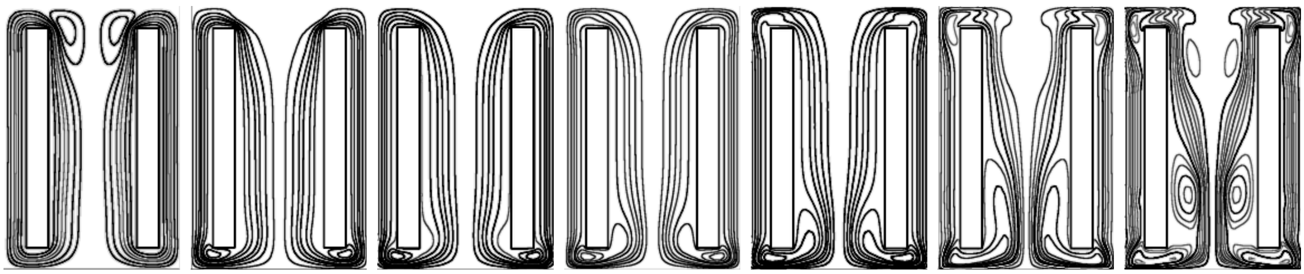


Figure 6. Efficiency of convective heat transfer due to thermomagnetic convection for different values of the Rayleigh number and magnetic field strength. The applied temperature gradient is oriented downwards, $\nabla T \downarrow$.



(a) (b) (c) (d) (e) (f) (g)

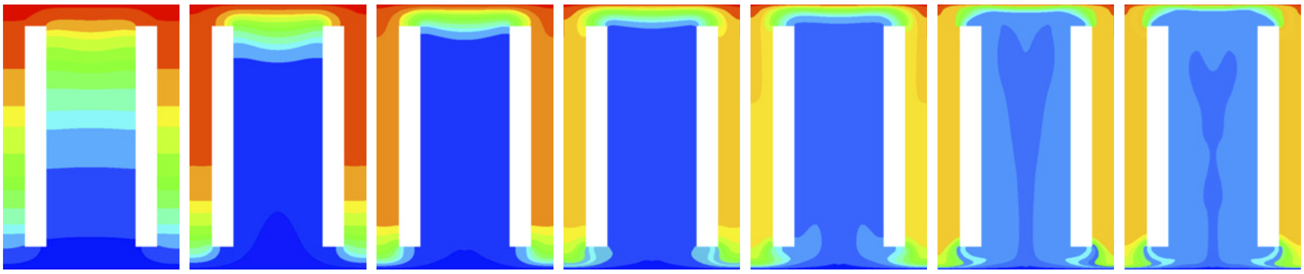
Figure 7. Stream function contours for $Ra = 10^4$ and different values of Rm/Ra: (a) $Rm/Ra = 10$, (b) $Rm/Ra = 20$, (c) $Rm/Ra = 30$, (d) $Rm/Ra = 50$, (e) $Rm/Ra = 100$, (f) $Rm/Ra = 500$, (g) $Rm/Ra = 1000$. The applied temperature gradient is oriented upwards, $\nabla T \uparrow$.

in figure 9. At the lower values of Rm/Ra the buoyancy effectively cancels the thermomagnetic convection. Further increasing the magnetic field, the thermomagnetic convection then becomes dominant. At higher values of Rm/Ra, within the region which interests us most, the efficiency of the heat transfer is approximately the same as in the case of downwards directed temperature gradient. The region where the buoyancy plays an important role is relatively narrow, which insures that the convection efficiency is orientation independent, unlike for the classical thermosyphons.

4. Conclusions

(i) The performed numerical calculations using the simplified set-up have shown that the efficiency of the convective heat transfer can be augmented greatly by the magnetic field. The on-axis velocity of the flow easily reaches 1 cm s^{-1} within the studied range of parameters for a thermosyphon with dimensions 2 cm $\times$ 6 cm.

(ii) The acquired data show promising possibilities for the experimental realization of the set-up, currently being



(a) (b) (c) (d) (e) (f) (g)

Figure 8. Temperature distribution contours for $Ra = 10^4$ and different values of Rm/Ra: (a) $Rm/Ra = 10$, (b) $Rm/Ra = 20$, (c) $Rm/Ra = 30$, (d) $Rm/Ra = 50$, (e) $Rm/Ra = 100$, (f) $Rm/Ra = 500$, (g) $Rm/Ra = 1000$. The applied temperature gradient is oriented upwards, $\nabla T \uparrow$.

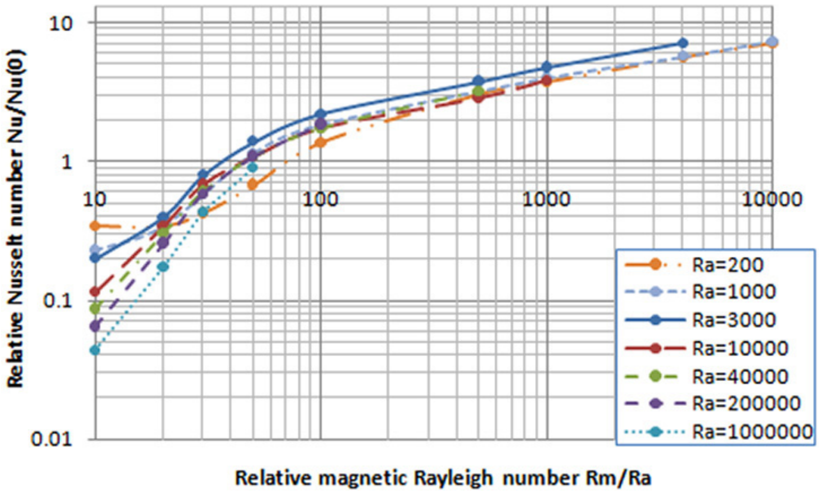


Figure 9. Efficiency of the convective heat transfer due to the thermomagnetic convection for different values of the Rayleigh number and magnetic field strength. The applied temperature gradient is oriented upwards, $\nabla T \uparrow$.

prepared. A solenoid, however, is very impractical for the purpose of generating a sufficiently strong magnetic field. Therefore, permanent magnets with saturation field 1.5 T will be used to provide the appropriate magnetic pressure. The distribution of the magnetic force will effectively be the same as studied here.

Acknowledgments

The work is supported by the European Regional Development Foundation, ERAF Latvia, project ERAF/CFLA/05/APK/2.5.1/000002/002.

References

[1] Blums E, Cebers A and Maiorov M M 1997 *Magnetic Fluids* (Berlin: Walter de Gryuter)

[2] Krakov M S and Nikiforov I V 2002 To the influence of uniform magnetic field on thermomagnetic convection in square cavity *J. Magn. Magn. Mater.* **252** 209–11

[3] Roache J P 1976 *Computational Fluid Dynamics* (Albequerque, NM: Hermosa)

[4] Mukhopadhyay A, Ganguly R, Sen S and Puri I 2005 A scaling analysis to characterize thermomagnetic convection *Int. J. Heat Mass Transfer* **48** 3485–92

[5] Filar P, Fornalik E, Tagawa T, Ozoe H and Szmyd J 2006 Numerical and experimental analyses of magnetic convection of paramagnetic fluid in a cylinder *J. Heat Transfer* **128** 183–91